

Enhancing the Inverted Topp-Leone Geometric Distribution: Properties and Applications in Computing

Saadia Tariq^{1*}, Junaid Talib¹, Umar Farooq², Abdul Hannan Khan³, Shahan Yamin Siddiqui⁴, and Muhammad Farrukh Khan⁴

¹School of Statistics, Minhaj University Lahore, Lahore, 54000, Pakistan.

²Department of Statistics, Government Associate College Depalpur, Okara, 56300, Pakistan.

³Department of Computer Science, Green International University, Lahore, 54000, Pakistan.

⁴Department of Computing, NASTP Institute of Information Technology, Lahore, 54000, Pakistan.

*Corresponding Author: Saadia Tariq. Email: Sadiasajad100@gmail.com

Received: March 09, 2024 Accepted: May 29, 2024 Published: June 01, 2024

Abstract: Statistics has developed a substantial interest in lifetime models, specifically within the domain of statistical inference. Practical domains such as computer science, medicine, engineering, biology science, management, and public health make extensive use of these models. Probability models find application in various domains, including game-winner prediction, team classification, winning margin evaluation, and likelihood of team victory. Recently, the model of mixed distribution has gained widespread recognition in the field of statistical data modelling. This paper aims a new two-parameter generalization of inverted Topp-Leone distribution. The new model, known as the Inverted Topp-Leone Geometric distribution, is created by mixing inverted Topp-Leone and geometric distributions. The quantile function, incomplete moments, ordinary moments, median, mode, mean residual life function, entropy, Shannon entropy, and mean deviation are some of the mathematical features of the new distribution that are obtained. Other characteristics include the mean deviation. The maximum likelihood approach is used to arrive at an estimate of the parameters of the model. Inverted (or inverse) distributions are advantageous for investigating further characteristics of the phenomenon. The behaviour of the parameter estimations is investigated by a Monte Carlo simulation. A practical computing application is provided to illustrate the new model's usefulness.

Keywords: Inverted Topp-Leone Distribution (ITLD); Geometric Distribution (GD); Maximum Likelihood Estimation (MLE).

1. Introduction

Several lifetime continuous distributions are derived to describe and predict the real-life phenomenon. However, there is always room for the development of new models that are more adaptable or more suited to certain real-world issues [1-2]. Probability distribution theory has also contributed to sample theory. Different Probability models are utilized to tackle social and real-world problems. With limited support, it displayed a continuous unimodal Topp-Leone distribution. This model was employed in subsequent statistical analyses as a substitute for the beta distribution when modelling life-time phenomena [3-5].

The hazard rate function has a bathtub shape, and the T-L distribution's density function has a J shape. When there are two separate causes of failure, mixtures of life-time distributions with the same parametrical form of distributions arise [6]. Given the significance of TL dispersion, several writers carried out related research. To investigate other aspects of the phenomenon, the inverted (or inverse) distributions are useful [7-9]. Studies in the fields of econometrics [10], engineering sciences [11], survey sampling [12], medical applications [13], and life testing challenges all make use of inverted distributions [14-17]. The right tail of the inverted TL distribution is rather lengthy.

The default rate value rises and gets closer to zero in the early stages as x increases. The goal is to conclude the proposed lot's acceptance or rejection because the units' actual median lifetime, exceeds the required lifetime [18-20]. When conducting a life test, it is customary to end it at a predetermined time and record the number of failures. It may also simulate bathtub and inverted J hazards, as well as hazards that are growing and diminishing simultaneously [21]. Being able to deal with a highly controllable and simple distribution of interest with an accurate closed-form CDF is another benefit [22-25]. This distribution is a very good fit for a variety of applications because of its strong requirements.

2. Materials and Methods

2.1. Derivation of New Model

Assume that "N" is a geometric random variable with a probability density function.

$$f(n, p) = p(N = n) = (1 - u) u^{n-1}, \quad n \in N, u \in (0,1)$$

Now

Let $z = \text{Min}\{z_1, z_2, z_3, \dots, z_N\}$ following a random sample taken from an inverted Topp-leone distribution.

$$G(z) = 1 - \left\{ \frac{(1 + 2y)^\theta}{(1 + y)^{2\theta}} \right\}; z \geq 0, \tag{1}$$

$$F(y)_{New} = \frac{F(y)}{1 - p(1 - F(y))}$$

The corresponding pdf is

$$f(y) = \frac{(1 - p) f(y)}{[1 - p \{1 - F(y)\}]^2}$$

By using Eq. (1), In the ITLG distribution, the unconditional CDF is considered to be

$$F_{ITLG}(y; \theta, p) = \frac{1 - \left\{ \frac{(1 + 2y)^\theta}{(1 + y)^{2\theta}} \right\}}{1 - p \left\{ \frac{(1 + 2y)^\theta}{(1 + y)^{2\theta}} \right\}}; z \geq 0, \theta > 0 \tag{2}$$

For $z > 0$, it is expressed as the probability density function (pdf) that corresponds to this statement.

$$f_{ITLG}(y; \theta, p) = \frac{2\theta(1 - p)y(1 + y)^{-2\theta-1}(1 + 2y)^{\theta-1}}{\left[1 - p \left\{ \frac{(1 + 2y)^\theta}{(1 + y)^{2\theta}} \right\} \right]^2} \tag{3}$$

For the given values of θ and p , some plots of the ITLG distribution are shown in Figures 1 and 2. In Figure 2, diagrams illustrate the HR function of ITLG Distribution.

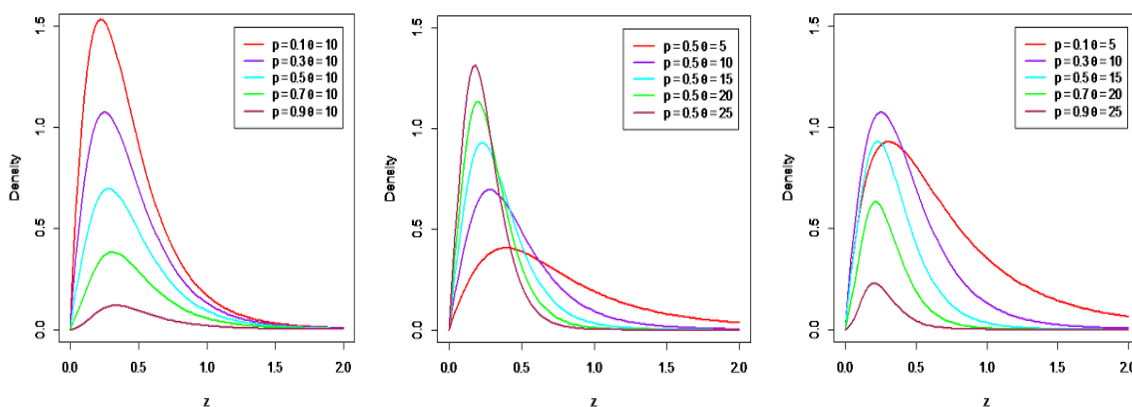


Figure 1. ITLG Distribution's PDF for Specific Parameter Values

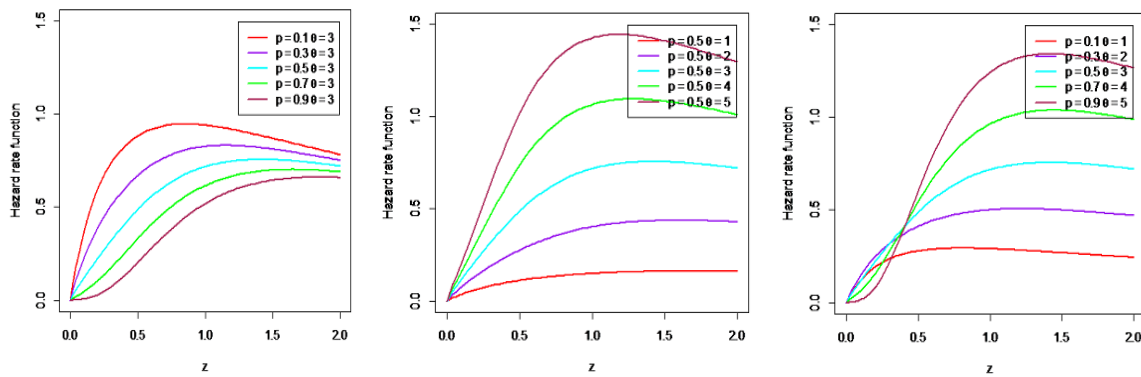


Figure 2. Shows the HF of the ITLG Distribution Parameter

2.2. Survival Function

$$S(y) = \frac{(1 - p) \left\{ \frac{(1 + 2y)^\theta}{(1 + y)^{2\theta}} \right\}}{1 - p \left\{ \frac{(1 + 2y)^\theta}{(1 + y)^{2\theta}} \right\}} \tag{4}$$

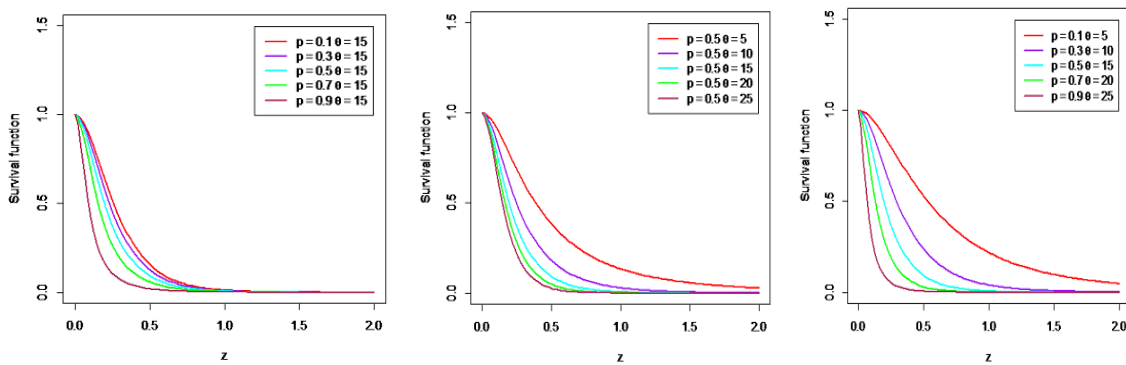


Figure 3. Certain parameter values for SF and ITLG Distribution

3. All Statistical Properties of the Proposed Distribution of ITLG

Linear Depiction for the ITLG Delivery System. We show a linear representation of the ITLG distribution and describe how to apply it to give a practical linear representation. This is one way to provide a mixed representation of the ITLG. Due to the complexity of the precise solution, numerical computations of skewness and kurtosis are performed.

Table 1. Parameter Combination Skewness in the ITLG Distribution

θ	P					
	0.1	0.25	0.45	0.5	0.75	0.95
5	3.5579	3.4799	3.7807	3.9308	5.5042	12.5168
6	2.7683	2.7189	3.0262	3.1689	4.5945	10.7004
7	2.3464	2.3152	2.6315	2.7716	4.1269	9.7752
8	2.7683	2.7189	3.0262	3.1689	4.5945	10.7004
9	2.3464	2.3152	2.6315	2.7716	4.1269	9.7752

Mode: As a result of Equation (4), the rth moment of z is calculated.

$$E(y^r) = \int_0^\infty y^r f(y) dy \tag{5}$$

Mean: The formula for calculating the mean of the ITLG distribution is as follows:

$$\mu'_1 = 2\theta(1 - p) \sum_{k=0}^{\infty} \sum_{t=0}^{\infty} \phi_{k,t} \beta(3 + t, 2\theta(k + 1) - \theta(k + 1) - 1) \tag{6}$$

Median: The median of the ITLG distribution is given as

$$(1 + z)^{2\theta}(1 - 0.5) + (1 + 2z)^\theta(P(0.5) - 1) = 0 \tag{7}$$

A description of the manner of the ITLG distribution is as follows:

$$[1 - 2\theta y^2 - 2y^2][(1 + y)^{2\theta} - p(1 + 2y)^\theta] = 4p\theta y^2(1 + 2y)^\theta \tag{8}$$

The definition of the mgf of the ITLG distribution is as follows:

$$M_z(t) = E(e^{tz}) = \int_0^\infty e^{tz} f(z) dz$$

$$M_z(t) = 2\theta(1 - p) \sum_{r=0}^{\infty} \sum_{k=0}^{\infty} \sum_{t=0}^{\infty} \phi_{k,t,r} \beta(r + t + 2, 2\theta(k + 1) - \theta(k + 1) - r)$$

Where

$$\phi_{k,t,r} = \frac{t^r \Gamma(2 + k)}{r! k!} p^k \binom{\theta(k + 1) - 1}{t}$$

Mainly because the computation of variance is challenging and complex. Consequently, to compute the variance in Table 3, we make use of a numerical estimation approach.

Table 2. The Variation of the ITLG Distribution Determined by the Combination of Parameters

Θ	P					
	0.1	0.25	0.45	0.5	0.75	0.95
5	0.5252	0.5081	0.4352	0.4089	0.2346	0.0511
6	0.3302	0.326	0.284	0.2677	0.1553	0.034
7	0.2316	0.2321	0.2048	0.1934	0.1131	0.0248
8	0.1741	0.1766	0.1573	0.1488	0.0876	0.0193
9	0.1372	0.1406	0.1262	0.1195	0.0706	0.0156

3.1. Cumulative HRF

$$H(y) = \int_{-\infty}^x h(u) du = -\log [1 - F(y)] = -\log(S(y))$$

$$= \log \left[1 - p \left\{ \frac{(1 + 2y)^\theta}{(1 + y)^{2\theta}} \right\} \right] - \log \left[(1 - p) \left\{ \frac{(1 + 2y)^\theta}{(1 + y)^{2\theta}} \right\} \right]$$

3.2. Mean Deviation

$$M.D(y) = \int_0^\infty |y - \mu| f(y) dz$$

$$M.D(z) = 2\mu \frac{1 - \left\{ \frac{(1 + 2y)^\theta}{(1 + z)^{2\theta}} \right\}}{1 - p \left\{ \frac{(1 + 2y)^\theta}{(1 + y)^{2\theta}} \right\}} - 4\theta(1 - p) \sum_{k=0}^{\infty} \sum_{t=0}^{\infty} \phi_{k,t} \beta_{\frac{\mu}{1-\mu}}(t + 3, 2\theta(k + 1) - \theta(k + 1) - 1)$$

3.3. Shannon Entropy

Entropy is an information metric that provides a quantified range of probable outcomes. Shannon established it in 1948. It is a measure of the average probable outcomes of a random variable [26-27].

It represents the mean of all conceivable outcomes for a random variable. Entropy reaches its maximum indicating that there isn't any uncertainty in its characterization [28].

Given an uncontrolled variable Z, a set z serving as its support, and the probability density function f(z), the Shannon's entropy of Z is

$$H_z = - \int_{-\infty}^{\infty} f(y) \log f(y) dz$$

Integrals' presence is one of our main requirements. Thus, the ITLG random variable Z's differential is Rényi Entropy.

3.4. Rényi Entropy

The Rényi entropy is widely used in computer science, econometrics, statistical inference, classification, and identification of statistical problems [29]. Z is given the Rényi entropy as though it adhered to the ITLG distribution.

$$= (1 - p)^v (2\theta)^v \sum_{k=0}^{\infty} \sum_{t=0}^{\infty} \phi_{k,t} \beta(v + t + 1, v(\theta + 2) + \theta k - v - 1)$$

Let Z be the parameter that corresponds to the observed distribution and let $p, (\theta)^z$ be the dimension of the parametric matrix. By [30], the sample LF is computed as follows:

For each sample size, the tests were run 10,000 times.

Tables 4, 5, and 6 display the results of the MC simulation. Calculations are made for the variance of estimated parameters, biases, mean of estimated characteristics, and mean square error [31]. These results are grounded in the predicted first-degree asymptotic theory, which states that bias and MSE decrease to zero as the sample size increases.

Table 3. Absolute Bias, Variance and MSE at $p = 0.5, \theta = 0.5$

Random Sample	30		100		300		500	
Parameters	p	Θ	p	θ	P	θ	p	θ
Estimate	0.4939	0.5364	0.4977	0.5109	0.4992	0.5042	0.4994	0.5019
Bias	0.006	0.0364	0.0022	0.0109	0.0007	0.0042	0.0002	0.0019
Variance	0.003	0.0226	0.0008	0.0056	0.0002	0.0018	0.0001	0.001
Mean Square Error	0.003	0.0212	0.0008	0.0057	0.0002	0.0018	0.0001	0.001

Table 4. Absolute Bias, variance and MSE $p = 0.5, \theta = 2$

Random Sample	30		100		300		500	
Parameters	p	θ	p	θ	p	θ	p	θ
Estimate	0.494	2.1387	0.4986	2.0395	0.4992	2.0139	0.4993	2.0095
Bias	0.0059	0.1387	0.0031	0.0395	0.0007	0.0139	0.0006	0.0095
Variance	0.003	0.3483	0.0008	0.088	0.0002	0.0289	0.0001	0.0171
Mean Square Error	0.0031	0.3675	0.0008	0.0895	0.0002	0.0291	0.0001	0.0172

Table 5. Bias, Variance, and MSE $p=0.9, =1.9$

Random Sample	30		100		300		500	
Parameters	p	Θ	p	θ	p	θ	p	θ
Estimate	0.8988	2.2729	0.8995	2.0031	0.8998	1.9315	0.8999	1.9179
Bias	0.0011	0.3729	0.0004	0.1031	0.0001	0.0315	0	0.0179
Variance	0.0001	1.6123	0	0.2409	0	0.0625	0	0.0347
Mean Square Error	0.0001	1.7514	0	0.2516	0	0.0635	0	0.035

4. Practical Application

If any outliers or medians are present in a data collection, boxplots are typically employed to demonstrate them. The boxplot shows that there is good agreement between the data points in the overall plot of the breaking stress of carbon fibers.

3.70, 2.740, 2.730, 3.110, 3.270, 2.870, 4.420, 2.410, 3.190, 3.280, 3.090, 1.870, 3.750, 2.430, 2.950, 2.960, 2.30, 2.670, 3.390, 2.81000, 4.2000, 3.310, 3.310, 2.850, 3.150, 2.350, 2.550, 2.810, 2.770, 2.170, 1.410, 3.680, 2.970, 2.760, 4.910, 3.680, 3.190, 1.5700, 0.8010, 1.590, 2.000, 1.220, 2.170, 1.170, 5.080, 3.510, 2.170, 1.690, 1.8400, 0.390, 3.680, 1.610, 2.790, 4.70, 1.570, 1.080, 2.030000, 1.890, 2.880, 2.820, 2.50, 3.60, 1.4700, 3.110, 3.220, 1.690, 3.150, 4.90, 2.970, 3.390, 2.930, 3.220, 3.330, 2.550, 2.560, 3.56000, 2.590, 2.380, 2.830, 1.920, 1.360, 0.980, 1.840, 1.590, 5.560, 1.7300, 1.120, 1.710, 2.480, 1.180, 1.250, 4.380, 2.480, 0.85000, 2.030, 1.80, 1.610, 2.120, 2.050, 3.650

Table 6: Descriptive Measures

X- min	Q_1	\tilde{X}	\bar{X}	Q3	X- max	S2
0.39000	1.83000	2.67500	2.60100	3.19700	5.56000	1.042100

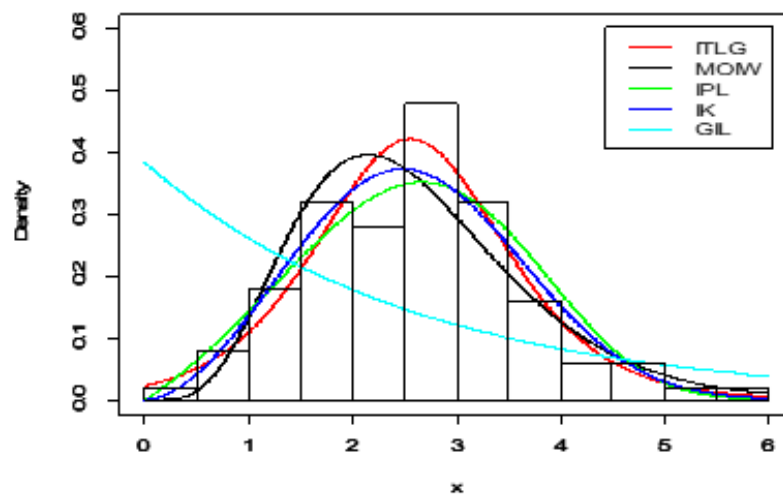


Figure 4. Estimated Densities of Data

Additionally, Figures 4 and 5 provide plots of fitted densities and empirical curve fits versus the MOIW, IPL, IK, and GIL distributions. These figures demonstrate that the ITLG distribution offers the most accurate representation of the actual data.

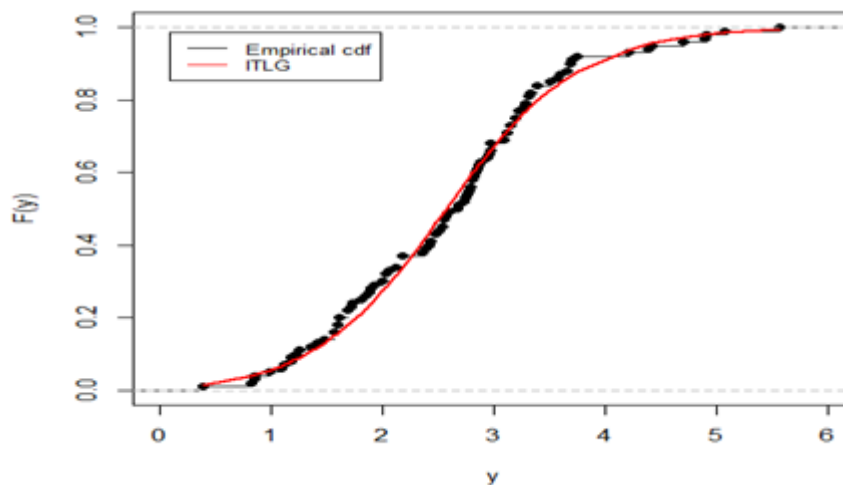


Figure 5. Empirical CDF Curves for MOE Distribution.

Table 7: MLEs of the Dataset

Model	Estimates of (Standard Deviations)		
ITLG	$\theta = 71.4130$ (3.15030)	$p = 0.66520$ (0.15340)	-

MOIW	$\alpha = 5.74940$ (0.79520)	$\beta = 2.23700$ (0.000800)	$\theta =$ 0.08970 (0.01160)
IPL	$\alpha = 1.98740$ (0.15660)	$\beta = 0.23170$ (0.00870)	$\theta =$ 0.52860 (0.32670)
IK	$\alpha = 2.75190$ (0.21130)	$\beta = 2.92270$ (0.11190)	-
GIL	$\alpha = 0.38420$ (0.03840)	$\theta = 1.26540$ (0.24170)	-

Table 8. Goodness of Fit Quantities of Dataset

Model	ITLG	MOIW	IPL	IK	GIL
-2logL	285.24	288.23	287.48	286.28	391.12
AIC	291.24	296.23	293.48	290.28	395.12
BIC	299.06	306.65	301.29	298.49	400.33
HQIC	294.41	300.44	296.64	294.91	397.22
K-S	0.0539	0.0966	0.088	0.0823	0.2521
MVS	0.0631	0.0841	0.0783	0.0724	0.1934
AD	0.413	0.612	0.598	0.561	0.92
p-value	0.9326	0.7981	0.8124	0.8329	0.3691

Based on an evaluation of all the models mentioned above, it is found that, for the given real-world data set, the ITLG distribution outperforms all other inverted distribution

5. Conclusion

The aforementioned table displays the values of ML estimates along with the six information requirements. The ITLG distribution provides the best fit out of MOIW, IPL, IK, and GIL. Compared to the distributions of MOIW, IPL, IK, and GIL, ITLG exhibits lower values for the aforementioned criterion. Final Thoughts on an Application. We illustrate the application of the inverted TLG distribution using real-world data and contrast it with MOIW, IPL, IK, and GIL models. The features are predicted using the MLE technique. The performance of MLE is assessed by simulation experiments. The comparison is made using a range of goodness of fit criteria and graphical analysis.

Funding: This study did not receive any money from outside sources.

Conflicts of Interest: The authors declare that they do not have any conflicts of interest to disclose concerning the current research.

References

1. Muhammad, M., Liu, L., Abba, B., Muhammad, I., Bouchane, M., Zhang, H., & Musa, S. (2023). A new extension of the Topp–Leone-family of models with applications to real data. *Annals of Data Science*, 10(1), 225-250.
2. El-Saeed, A. R., Hassan, A. S., Elharoun, N. M., Al Mutairi, A., Khashab, R. H., & Nassr, S. G. (2023). A class of power inverted Topp-Leone distribution: Properties, different estimation methods & applications. *Journal of Radiation Research and Applied Sciences*, 16(4), 100643.
3. Abushal, T. A., Hassan, A. S., El-Saeed, A. R. & Nassr, S. G. (2021). Power Inverted Topp–Leone Distribution in Acceptance Sampling Plans. *CMC-COMPUTERS MATERIALS & CONTINUA*, 67(1), 991-1011.
4. Elgarhy, M., Alsadat, N., Hassan, A. S., Chesneau, C., & Abdel-Hamid, A. H. (2023). A new asymmetric modified Topp–Leone distribution: Classical and Bayesian estimations under progressive type-II censored data with applications. *Symmetry*, 15(7), 1396.
5. Al-Dayian, G. R., El-Helbawy, A. A., Refaey, R. M. & Behairy, S. M. (2021). Bayesian Estimation and Prediction Based on Constant Stress-Partially Accelerated Life Testing for Topp Leone-Inverted Kumaraswamy Distribution. *Journal of Advances in Mathematics and Computer Science*, 11-32.
6. Al-Marzouki, S., Jamal, F., Chesneau, C. & Elgarhy, M. (2020). Type II Topp Leone power Lomax distribution with applications. *Mathematics*, 8(1), 4.
7. Almetwally, E. M. (2021). The Odd Weibull Inverse Topp–Leone Distribution with Applications to COVID-19 Data. *Annals of Data Science*, 1-20.
8. Almetwally, E. M., Alharbi, R., Alnagar, D. & Hafez, E. H. (2021). A new inverted topp-leone distribution: applications to the COVID-19 mortality rate in two different countries. *Axioms*, 10(1), 25.
9. Al-Saiary, Z. A. & Bakoban, R. A. (2020). The Topp-Leone Generalized Inverted Exponential Distribution with Real Data Applications. *Entropy*, 22(10), 1144.
10. Oluyede, B., Dingalo, N., & Chipepa, F. (2023). The Topp-Leone-Harris-G family of distributions with applications. *International Journal of Mathematics in Operational Research*, 24(4), 554-582.
11. Arshad, M. & Azhad Jamal, Q. (2019). Interval estimation for Topp-Leone generated family of distributions based on dual generalized order statistics. *American Journal of Mathematical and Management Sciences*, 38(3), 227-240.
12. Arshad, M. & Jamal, Q. A. (2019). Statistical inference for Topp–Leone-generated family of distributions based on records. *Journal of Statistical Theory and Applications*, 18(1), 65-78.
13. Bantan, R. A., Jamal, F., Chesneau, C. & Elgarhy, M. (2019). A New Power Topp–Leone Generated Family of Distributions with Applications. *Entropy*, 21(12), 1177.
14. Bantan, R. A., Jamal, F., Chesneau, C. & Elgarhy, M. (2020). Type II Power Topp-Leone generated family of distributions with statistical inference and applications. *Symmetry*, 12(1), 75.
15. Behariy, S. M., Refaey, R. M., EL-Helbawy, A. A. & AL-Dayian, G. R. (2020). Topp Leone-inverted Kumaraswamy distribution: Properties, estimation and prediction. *Journal of Applied Probability and Statistics*, 15(3), 93-118.
16. Eldeeb, A. S., Ahsan-Ul-Haq, M. & Babar, A. (2021). A Discrete Analog of Inverted Topp-Leone Distribution: Properties, Estimation and Applications. *International Journal of Analysis and Applications*, 19(5), 695-708.
17. Elgarhy, M., Arslan Nasir, M., Jamal, F. & Ozel, G. (2018). The type II Topp-Leone generated family of distributions: Properties and applications. *Journal of Statistics and Management Systems*, 21(8), 1529-1551.
18. Hassan, A. S., Almetwally, E. M. & Ibrahim, G. M. (2021). Kumaraswamy Inverted Topp–Leone Distribution with Applications to COVID-19 Data. *CMC-COMPUTERS MATERIALS & CONTINUA*, 68(1), 337-358.
19. Khan, M. J. S. & Iqar, S. (2019). On moments of dual generalized order statistics from Topp-Leone distribution. *Communications in Statistics-Theory and Methods*, 48(3), 479-492.
20. Oguntunde, P. E., Khaleel, M. A., Okagbue, H. I. & Odetunmbi, O. A. (2019). The Topp–Leone Lomax (TLLO) distribution with applications to airborne communication transceiver dataset. *Wireless Personal Communications*, 109(1), 349-360.
21. Oluyede, B., Chipepa, F. & Wanduku, D. (2021). The odd Weibull–Topp–Leone–G power series family of distributions: model, properties and applications. *J. of Nonlinear Sciences and Applications*, 14, 268-286.
22. Rasekhi, M., Alizadeh, M. & Hamedani, G. G. (2018). The Kumaraswamy Weibull Geometric Distribution with Applications. *Pakistan Journal of Statistics and Operation Research*, 347-366.
23. Reyad, H., Jamal, F., Othman, S. & Yahia, N. (2019). The Topp Leone generalized inverted Kumaraswamy distribution: Properties and applications. *Asian Research Journal of Mathematics*, 1-15.
24. Rezaei, S., Sadr, B. B., Alizadeh, M. & Nadarajah, S. (2017). Topp–Leone generated family of distributions: Properties and applications. *Communications in Statistics-Theory and Methods*, 46(6), 2893-2909.
25. Sule, I., Doguwa, S. I., Isah, A. & Jibril, H. M. (2020). The topp leone kumaraswamy-g family of distributions with applications to cancer disease data. *Journal of Biostatistics and Epidemiology*.
26. Khan, Z., Noman, M., Jan, S. T., & Khan, A. D. (2023). Systematic investigation of the impact of kesterite and zinc based charge transport layers on the device performance and optoelectronic properties of ecofriendly tin (Sn) based perovskite solar cells. *Solar Energy*, 257, 58-87.

27. Khan, A. H., Khan, M. A., Abbas, S., Siddiqui, S. Y., Saeed, M. A., Alfayad, M., & Elmitwally, N. S. (2021). Simulation, modeling, and optimization of intelligent kidney disease predication empowered with computational intelligence approaches. *Computers, Materials & Continua*, 67(2), 1399-1412.
28. Khan, A. H., Abbas, S., Khan, M. A., Farooq, U., Khan, W. A., Siddiqui, S. Y., & Ahmad, A. (2022). Intelligent model for brain tumor identification using deep learning. *Applied Computational Intelligence and Soft Computing*, 2022, 1-10.
29. Khan, A. H., Hasnain, S. A., Siddiqui, S. Y., Irshad, M. S., Sajid, M., & Iqbal, S. (2020). Analytical Method to Compute the Cloud Computing Data Security Issues by Using Encryption Algorithms. *EAI Endorsed Transactions on Scalable Information Systems*, 7(28), e3-e3.
30. Nawaz, A., Atif, M., Khan, A., Siddique, M., Ali, N., Naz, F., ... & Boczkaj, G. (2023). Solar light driven degradation of textile dye contaminants for wastewater treatment—studies of novel polycationic selenide photocatalyst and process optimization by response surface methodology desirability factor. *Chemosphere*, 328, 138476.
31. ZeinEldin, R. A., Jamal, F., Chesneau, C. & Elgarhy, M. (2019). Type II Topp–Leone Inverted Kumaraswamy Distribution with Statistical Inference and Applications *Symmetry*, 11(12), 1459.