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Floating Non-Parametric Control Charts for Process Parameter Using Computation

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Abstract: When there are doubts about the intended quality attributes parametric distribution, nonparametric control charts are frequently utilized. The majority of research in the literature on statistical process control (SPC) employed parametric techniques, wherein the functional connection remains unchanged across in-control (IC) and out-of-control (OC) scenarios. Nonparametric profiles are highly prevalent and have a distinct functional relationship under OC circumstances. Control charts are an excellent resource for analyzing variability. Control charts with a progressive mean and cumulative sum are useful for process monitoring. The CUSUM chart presents data from the current and prior samples, as opposed to calculating the subgroup mean separately. With a single sample, this graph performs well. By adjusting the smoothing value after each sample, the progressive mean statistic is a variant of the EWMA statistic. Every sample in the PM charts is given equal weight, taking into account both the current and all prior samples. A floating nonparametric (FNPCUSUM) control chart was our suggestion in this situation. A comparison was made between the proposed chart and earlier charts that were created with varying run lengths. The effectiveness of utilizing computation to create floating nonparametric control charts is demonstrated by this comparison. The comparison showed that, especially for minor to moderate changes in process location, the suggested control chart performed better than the other competing control charts. Finally, to demonstrate the effectiveness of the suggested control chart, a real-world application is also provided for quality practitioners.

Keywords: Progressive Mean; CUSUM Charts; Nonparametric; Average Run Length.

1. Introduction

A graphical tool for monitoring the actions that occur throughout a manufacturing process is the control chart. It is an important statistical process control (SPC) procedure. These charts allow us to determine whether a machine or device replacement is necessary to achieve quality standards. The Process of using statistical analytic techniques to forecast the characteristics, features, or standards of the good that will be produced is known as statistical quality control or SQC. Statistical quality control has become an essential component of the production process, which aims to improve product quality and productivity. There are two approaches to supply chain quality control (SQC) that are used to monitor and maintain product quality. One technique is sampling acceptance, often known as product control, which involves inspecting the final product and determining whether it complies with requirements. Plans for acceptability sampling the final product were created by Harold Dodge in 1925 [1].

Control charts are sometimes referred to as process behavior charts. Statistical process control is used to determine if a business or industrial Process is under statistical control. Memory type and memoryless control charts are two different varieties. Memory-less control charts are typically employed for significant

changes in the process parameter. However, modest and moderate process shifts are handled using memory-type control charts. These control charts are used in a wide range of sectors, including engineering, scientific testing, education, and health. They are not limited to the manufacturing business. New concepts for employing the progressive mean (PM) statistic in statistical process control to track Process mean and dispersion have been put forth [2].

The PM chart is a valuable tool for tracking minor and moderate shifts. In comparison to standard Shewhart, EWMA, CUSUM, and some other variations of them, it demonstrates the enhanced performance of PM charts. PM charts are designed in a way that uses both historical and present data. When parametric procedures fail to meet the distributional constraints, nonparametric techniques are usually employed for data analysis. Statistical methods that yield results that are independent of all or most of the assumptions made regarding the distributional form of the data are known as nonparametric methods. Since real-world data is typically clumped or non-linear and is not normally distributed, nonparametric approaches are more adaptable in these contexts. It is believed that nonparametric approaches are less prone to misuse and misinterpretation due to their robustness and simplicity.

These techniques are primarily applied to populations that exhibit rank ordering, e.g., moving reviews and ratings, reviewing eateries, etc. Large sample sizes of data result in a significant loss of statistical power when using nonparametric approaches. A number of researchers have worked in this area. Raji, Lee, Riaz, Abujiya, and Abbas [3] tested the system's robustness using the mean estimator and looked into the multivariate Shewhart chart as a location parameter monitor. They also investigate the system in a variety of process scenarios using a few different trustworthy parametric estimators. An updated double progressive mean (DPM) control chart is presented by Riaz, Abid, Abbas, and Nazir [4]. In the previous DPM control chart, variance expression is missing this information. Ali, Abbas, Nazir, Riaz, Zhang & Li [5] proposed a chart based on sign and arcsine test data; a modified nonparametric exponentially weighted moving average chart is created below the progressive scenario. Abbas Nazir, Riaz, Abid, and Akhtar [6] offered a new NP double progressive mean chart that uses sign statistics to identify even the smallest changes in the process location. Abbas, Ali, Nazir, Riaz, Li & Zhang [7] created, using the sign test statistic and arcsine transformation, exponentially weighted moving average and cumulative sum control charts under zero-state and steady-state conditions at head-to-head optimal design parameter selections. When most control charts are constructed to use a single sample scheme, Nawaz, Azam and Aslam [8] recommend a recurring sampling strategy to create new control charts.

They use exponentially weighted moving averages and double exponentially moving weighted averages to monitor shifts in the Process. The triple exponentially weighted moving average (TEWMA) control chart, which Alvizakos, Chatterjee, and Koukouvino [9] suggested a new chart used to track the position parameter of an unknown continuous distribution. The sign statistic serves as the foundation for it. When it comes to identifying deviations in the process location for heavy-tailed and skewed distributions, Abbas, Nazir, Akhtar, Abid & Riaz [10] proposed NPPM-SR chart demonstrates a robust, incontrol performance.

Riaz, Abid, Abbas, and Nazir [11] present an updated double progressive mean (DPM) control chart. The previous DPM control chart's variance expression is missing this information. Together with the previously described variance's accurate variation, they also provide its corresponding limitations. The free of distributions (nonparametric) statistical quality-controlled chart for monitoring the centering Process is provided by Bakir [12]. It's a control chart of the Shewhart types, made with the sign ranks of the variable groupings as the basis. Control charts with a cumulative sum and progressive mean are useful for process monitoring. Das & Bhattacharya [13] propose a method based on the squared rank test for Conover variance. They also evaluate its efficacy in terms of its performance in comparison to in-control ARL and its capacity to recognize variations in variability.

[14] Introduced the signed-rank statistic-based distribution-free mixed EWMA-MA control chart, which is useful for effectively identifying changes in the process location.

This study suggested Floating Non-Parametric (FNPCUSUM) control charts for process optimization in this situation. A comparison was made between the suggested chart and other suggested charts with varying run lengths.

2. Proposed Methodology

Process variability can be examined with the cumulative sum control chart, just like it can with other The primary objective of the CUSUM control chart is to maintain process location. control charts. "Cumulative Sum" refers to the overall difference between the target and the mean of each sample result or subgroup. When distributional requirements for parametric methods are not satisfied, nonparametric approaches are most usually used to analyze data.

Nonparametric statistical approaches are defined as those whose outcomes are independent of all or most of the assumptions made about the distributional form of the data. Nonparametric control charts are commonly utilized when uncertainties exist regarding the parametric distribution of the intended quality parameter. Control charts are a great tool for examining variability. Control charts with a progressive mean and cumulative sum are useful for process monitoring. It suggested Estimation of Floating Nonparametric (FNPCUSUM) Process Control Charts in this regard. The suggested chart was contrasted with earlier charts that were suggested but used different run lengths. This comparison illustrates the chart's effectiveness. The progressive means provides the foundation for the process monitoring data used in this investigation. The progressive mean is the average of the sample values gathered over a period. Assume that a distribution f(Y) follows a quality characteristic Y for monitoring. Size n, Y1, Y2... Should be used to create samples for this distribution. According to its definition, the progressive mean statistic is:

$$PM_{n} = \frac{Y_{1} + Y_{2} + Y_{3} + \dots + Y_{n}}{n}$$
(i)
= $\frac{\sum_{i}^{n} Y_{i}}{n}$ (ii)

This statistical measure is an unbiased estimator of variance σ^2 and $\mu = \mu^0$. 2.1. Nonparametric Progressive Mean

Let us assume that 'n' size samples obtained from a process have the intended value μ are Y1, Y2,..., *Yn.* In this case, p is defined as the probability that Y will be bigger than μ , or p = pr (Y > μ).

If $p \neq P0$, the Process is out of control; if p = P0, the Process is said to be in control. In this, we want to monitor the stability of p in order to regulate the location parameter with the reference of 0 [15]. That showed M= $\sum_{i=1}^{n} I_i$ that is distributed binomially with parameters (n, *p*0), where the value of I_i is equal to 1 when $Y_i > \mu$ and 0 in other cases, the binomial distribution can be changed into a normal distribution by applying the arcsine transformation.

$$z = \frac{\sin^{-1} \sqrt{\frac{M}{N^{\sim}}} N(\sin^{-1} \sqrt{p_o}, \frac{1}{4n})}{\mu_z = \sin^{-1} \sqrt{p_o} and \sigma_z = \frac{1}{4n}}$$
(iii)

For point 't,' PM statistics are generated here using z.

$$PM_{t} = \frac{\sum_{j=1}^{t} z_{j}}{t}$$
 (iv)

2.2. The CUSUM Control Charts

=

Considering the quality parameter x has a required value of $\mu 0$. For sums from $\mu 0$ that are above the target, Tabular CUSUM uses c+, while for derivations from $\mu 0$ that are below the target, it uses c-. They are computed as:

$$c_i^+ = \max(0, x_i^- (\mu_0^+ K) + c_{i-1}^+) \quad (v)$$

$$c_i^- = \max(0, (\mu_0^- K) - x_i^+ c_{i-1}^-) \quad (vi)$$

The initial values in this case are c0 = 0. Where k is the K=k σ reference value. Additionally, there is the decision interval H. If c_i^+ and c_i^- are deemed out of control when they surpass

the Process, where $H=h\sigma$ in this scenario.

2.3. Suggested Nonparametric Floating CUSUM (FNPCUSUM) Control Charts

The monitoring statistics utilized in this work are nonparametric progressive mean statistics. Our innovative method of estimating floating nonparametric control charts for process parameters uses a CUSUM control chart as its plotting statistic. Charts employ the arcsine transforms. For the charts, the PM statistic is as

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(vii)

 $PM_t = \frac{\sum_{j=1}^t z_j}{t}$

Here $\mu_z = \sin^{-1} \sqrt{p_o}$ then $\sigma_z = \frac{1}{4n}$

Plotting statistics for the given charts are

$$Mp [t] = max (0,(w[t]-mu)-K+Mp[t-1])$$
(viii)

$$Mn[t] = max(0,-(w[t]-mu)-K+Mn[t-1])$$
(ix)

The symbol w[t] represents the standardized progressive mean of Yi at time t, where K is the reference value. The initial values of the statistic were set to be zero, meaning that Mp[t] = 0 and Mn[t] = 0. Let us define the two-sided CUSUM control chart with H+ representing the upper control limit (UCL) for Mp and H- representing the lower control limit (LCL) for Mn. Zero is the position of the middle line. In this case, we consider that the Process is out of control if Mn > H+ and Mn ≤ H-.

2.4. Steps for the Performance of Chart

Various ARL0's, like ARL0= 270 ARL0 = 500, are computed using different sample sizes of 5, 7, 10, and 20. The computational procedures for these calculations are described. Create an n-sized random sample with the parameter n and p = p0 = 0.5 using the binomial distribution.

- The data was transformed using the arcsine transformation, which is $z = \sin^{-1} \sqrt{\frac{M}{N}}$
- Proceed to calculate the progressive mean statistic by utilizing the expression found in (vii)
- The data is standardized using a z-score in order to facilitate smoothing. This yielded through μ = 0 and σ 2= 1.
- For the fixed value of k = 2.5 in expressions (viii) and (ix), respectively, different values of h were chosen for the pre-specified ARL0's for the computation control limits.
- The run length is the sample size at which the charting statistic deviates from the specified range.
- We continue sampling and repeat steps I, 2, 3, 4, and 5 until the plotted statistics fall in one of the two determining zones if Pm[i] < H and Mn[i] < H.
- Do the previously mentioned steps 100,000 times to calculate the in-control ARL. The outcomes and conclusions covered in Tables 1 and 2 are described by the data analysis. Using

ARL, we assess how well the procedure works using the floating nonparametric control chart estimation that is suggested. For varying values of h and on samples with sizes of 5, 7, 15, and 20, we fixed ARL0 \cong 270 and 500 for the values of p= 0.5 and k = 2.5. In order to make a comparison, we set ARL0 \cong 370 for values of h that range from 0.613 to 2.5 and n = 10, 15, and 20.

3. Performance Evaluation

The data analysis explains the outcomes and conclusions covered in the prior section. Through the use of ARL, we assess how well the proposed floating nonparametric control charts for the Process works.

	Table 1. The Run Length characteristics of the proposed chart when ARL ₀ = 270													
	$\mathbf{A}^{[]]}_{rlo} \approx 270$													
	p_o													
n ł	=2.5	0.05	0.15 0.	.25 0.3	0 0.35	5 0.40	0.45	0.50	0.55	0.60	0.65 0	.70 0.	75 0.8	5 0.95
5	h=30	6.78	8.81	12.19	15.02	19.8	29.15	60.9	273.84	60.58	29.29	19.62	14.97	12.13
							8.85	6.78						
7	h= 17	4.6	8 6.27	8.82	11.02	14.76	22.18	46.96	272.58	47.02	22.02	14.65	11.08	8.84
							6.23	4.68						
12	2 h=12	3.58	8 4.89	6.92	8.62	11.46	17.33	37.16	269.95	37.41	l 17.29	11.49	8.62	6.91
							4.89	3.59						

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20 h=9 2.5 3.4 4.78 5.9 7.79 11.57 24.26 272.69 24.3 11.6 7.71 5.88 4.76 3.39 2.5

Table 2. The Run Length characteristics of the proposed chart when ARL ₀ = 500
$A^{\text{IIII}}_{rlo} \approx 500$

	p_o															
n	k=2.5	0.05	5 0.15	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.85	0.95
5	h=98	12.4	15.72	20.92	25.32	32.18	45.33	87.83	502.58	87.95	45.76	32.27	25.24	20.97	15.69	12.42
7	h=65	9.08	11.78	16.05	19.48	24.89	35.59	68.83	500.18	68.92	35.47	24.87	19.43	16.03	11.76	9.09
12	h=48	7.00	9.26	12.67	15.34	19.69	28.06	54.87	503.2	54.62	28.02	19.62	15.36	12.65	9.26	7.00
20	h=36	4.95	6.56	8.81	10.61	13.46	18.98	36.22	501.91	1 36.12	2 19.21	13.48	10.61	8.84	6.56	4.95

In the preceding sample, with n = 5, 7, 10, and 20, we fixed the values of k at 2.5 ARL0 \cong 270 and 500 at *P*0 = 0.5. At shifts of 0.05, 0.15, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.85, and 0.95, we displayed the results at various values of h. It has been noted that when the sample size grows, the value of ARL0 falls. For instance, for n=20, ARL0 is reduced, and at the shift of p=0.95, *P*0 = 0.5. 3.1. Comparative Analysis

The performance of the suggested chart's ARLs in detecting changes in the process mean is compared with that of the mean chart [16] and the arcsine CUSUM chart [17] in terms of control shifts ARL0's set at 370.5 runs in Table III. Results show that recommended floating nonparametric control charts are more profound to both minor and significant variations in the process location than other charts. With a larger sample size, the suggested chart, however, fared better than the current one.

control chart's performance with AREO =570, p=0.015											
CHARTS		Mean	arcsine	FNPCU	Mea	arcsine	FNPCU	Mea	arcsine	FNPCU	
		chart	CUSU	SUM	n	CUSU	SUM	n	CUSU	SUM	
			M chart	control	char	М	control	ontrol chart		control	
					t	chart			chart		
n			10			15			20		
р	0.55	2936.8	166.1	54.85	810	104.7	35.54	609	85.4	28.1	
	0.613	370.5	370.5	373.47	370.	370.5	372.73	370.5	370.5	370.5	
					5						
	0.65	36251	204.3	66.28	617.	184	49.19	458.1	162.1	41.31	
					8						
	0.75	1048576	49.6	22.41	74.8	37.2	15.32	41.1	26.8	12.47	
	0.85	1.73E+0	15.8	13.92	11.4	11.5	9.33	5.7	8.4	7.67	
		8									

Table 3. Comparing the arcsine CUSUM chart and mean chart with the floating nonparametric control chart's performance with ARLo =370, p=0.613



Figure 1. Comparing the arc-sine CUSUM, FNPCUSUM, and Mean chart at n = 10 ARL = 370.5, p = 0.613







Figure 3. Comparing the control charts for arcsine CUSUM and FNPCUSUM at n = 20, ARL = 370.5, p = 0.613

To illustrate its detection capacity, a data set is produced and the recommended estimation of FNPCUSUM charts for the Process is applied [18] [19]. The following parameters are obtained for 30 samples of size 10: ARL0 = 370, and the expected mean in \overline{X} = 0.48, an estimated with p0 = 0.5, and C 0 + = C 0-= 0 for the ARLo, having k = 0.5 and H = 10.65, which didn't identify any shift in the Process [20] [21] [22].



Figure 4. Floating Parametric (FNPCUSUM) control chart for Process

When a process gets out of control, 8 extra samples of size 10 are introduced at p0= 0.65. Figure 5 illustrates how 32 to 34 exceed the arcsine bounds [20] [23]. On the other hand, the CUSUM mean chart does not display any out–of–control spots. Our proposed chart in Figure 5 shows that shifted at p0= 0.65 k = 2.5 h = 25 and H = ± 0.625 shows points at 5-9 fell above the UCL of the proposed chart, signaling that

the Process is not in control. It detects the out-of-control points in the beginning. However, the procedure was not found to have changed, according to the mean chart.







Figure 6. Nonparametric MEAN Chart

4. Conclusions

A control chart is considered effective if it has reduced out-of-control ARLs during particular shifts. Two primary chart kinds are available to monitor the process location: memory control charts such as EWMA and CUSUM and Shewhart-type control charts. While the latter are good at identifying tiny and intermediate shifts, the former are advised for bigger shifts. We have contrasted CUSUM mean control charts and arcsine control charts in our estimation of floating nonparametric control charts. According to the findings, the recommended chart may identify both small and major shifts in the Process mean. It has been observed that the suggested chart's detection capability is effective for both positive and negative shifts in *p*0. As the sample size n increases, the run length's ARL decreases. The chart's distribution of run lengths is positively skewed. An additional comparative analysis revealed that the suggested chart is more effective at identifying shifts when comparing sample sizes to identify changes in the Process mean. Control charts are implemented using a data set, and the results show that the suggested chart performs better than the others. According to the results and conclusions of a real-world application derived from a prior study, the suggested estimation of floating nonparametric control charts outperformed these charts.

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